

Fibonacci, quasicrystals and the beauty of flowers

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The appearance of Fibonacci sequences and the golden ratio in plant structures is one of the great outstanding puzzles of biology. Here I suggest that quasicrystals, which naturally pack in the golden ratio, may be ubiquitous in biological systems and introduce the golden ratio into plant phyllotaxy. The appearance of golden ratio-based structures as beautiful indicates that the golden ratio may play a role in the development of consciousness and lead to the aesthetic natural selection of flowering plants.

Fibonacci and the Golden Ratio

In mathematics and the arts, two quantities are in the golden ratio (τ or 1.6180339...) if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one. The golden ratio is thought to be aesthetically pleasing and thus appears often in art and architecture. The Fibonacci numbers are simply the arithmetic consequence of multiplying each number by τ , and rounding to the nearest integer or, in other words, the next number in the Fibonacci sequence is the sum of the previous two numbers in the sequence thus 1, 1, 2, 3, 5, 8, 13, 21, 34, ... The Fibonacci sequence was first discovered by Leonardo da Pisa (also known as Fibonacci) in 1202 and has puzzled scientists for centuries due to its frequent occurrence in nature, including in spiral molluscan shells and plants. The ubiquity of the Fibonacci sequence in nature seems to indicate that it arises through a process that is fundamental to life.

Fibonacci Phyllotaxy and Fractals

Phyllotaxy in plants has attracted attention for the repeated appearance of Fibonacci sequence phyllotactic patterns. Developmental processes in plants give rise to an almost constant golden divergence angle, constant plastochrome ratio, choice of parastichy numbers and prevalence of Fibonacci sequences to which these numbers belong. These patterns are particularly seen in the spiral form of flowers (Fig. 1A).¹ It has been suggested that this form of phyllotaxy has a selective advantage as it involves the least number of phyllotactic transitions during growth.² Various mechanisms have been suggested for the creation of Fibonacci sequence-based patterns in plants. These include active transport of auxin,³ buckling of the plant's tunica⁴ and the material properties of the cell wall.⁵ However, it seems that the Fibonacci sequence must arise in biological systems through a process common to animals and plants, not plant-specific. Here I propose that fractals and quasicrystals may contribute to the formation of these Fibonacci patterns.

Fibonacci fractal patterns grow through the addition of the previous two patterns in the sequence (Fig. 1B). Fibonacci fractal patterns at the subcellular level may in turn lead to the creation of macroscopic Fibonacci phyllotaxy. In plants, where higher numbers of the Fibonacci sequence appear macroscopically, presumably the lower numbers of the sequence are present at the cellular or subcellular level as part of the fractal progression. Microtubules may be involved and it is also likely that the cell wall and mechanical stress will be involved in coordinating development

Keywords: quasicrystal, Fibonacci, flower, golden ratio, fractal

Submitted: 09/22/12

Accepted: 10/01/12

<http://dx.doi.org/10.4161/psb.22417>

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Addendum to: Gardiner J. Insights into plant consciousness from neuroscience, physics and mathematics: A role for quasicrystals? *Plant Signal Behav* 2012; 7:1049-55; PMID:22899055; <http://dx.doi.org/10.4161/psb.21325>.

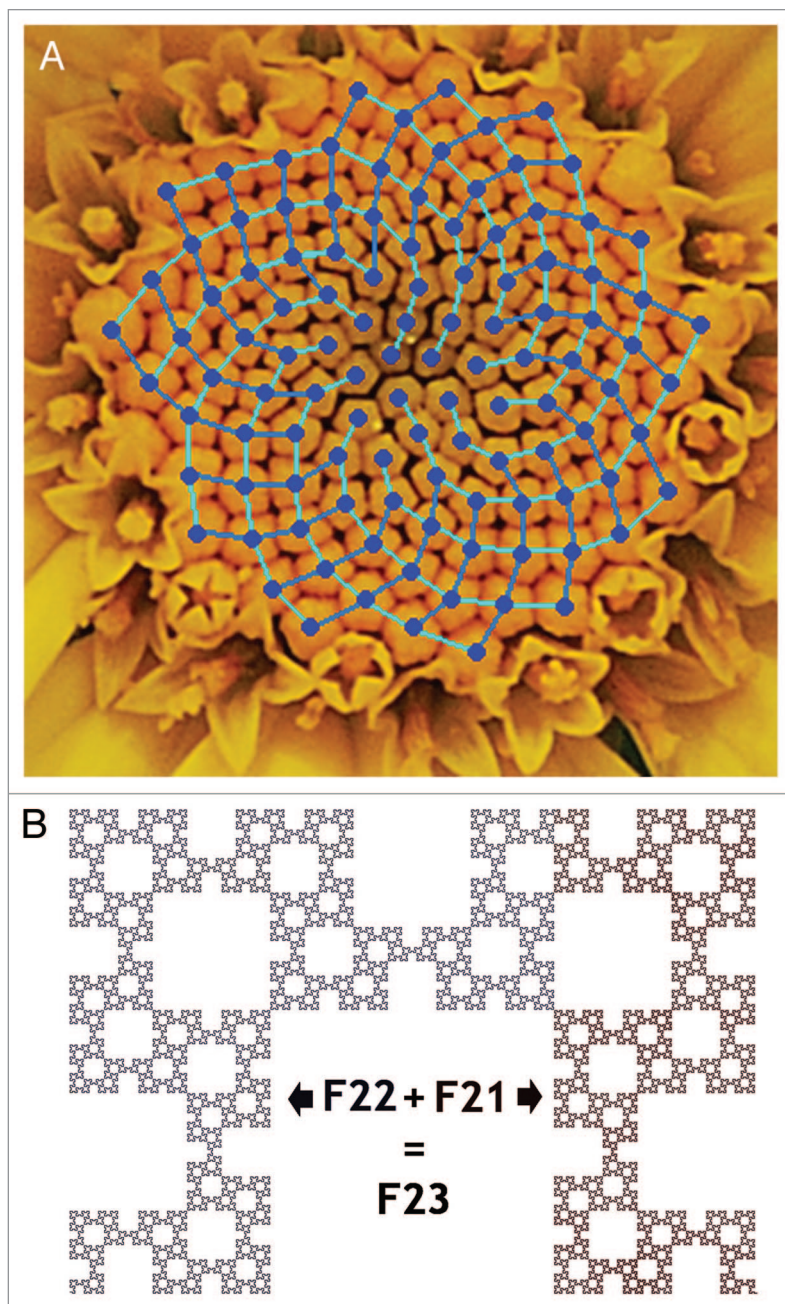


Figure 1. (A) A flower showing Fibonacci organization. Spirals of florets are in groups of both 13 (green) and 21 (blue), which are consecutive Fibonacci sequence numbers. (B) A Fibonacci fractal showing that the addition of the previous two objects in the sequence gives the next object. Perhaps similar to the flower in Figure 1A?

across organs. Evidence for this suggestion includes the tight coupling of microtubules and PIN1 auxin transporter in the shoot apical meristem, downstream of a biomechanical regulator.⁶ Intriguingly, mutation in a DNA topoisomerase I gene, involved in the creation of fractal DNA superhelices *in planta*, causes defects in helical plant phyllotaxy.⁷

Do Quasicrystals Participate in Flower Fibonacci Phyllotaxy?

If there are cellular and subcellular Fibonacci fractals in plants, what form do they take? Quasicrystals are aperiodic formations with forbidden non-crystallographic symmetry. Quasicrystals were first reported in 1984,⁸ in this case forbidden

icosahedral symmetry from aluminum-manganese alloys. In a previous paper I suggested that pentagonal ribosome formations in *Pteridium aquilinum* fertilized eggs may be an example of a quasicrystal from a living organism.⁹ In fact biological quasicrystals may be common. A quasicrystalline phase formed in aqueous dispersions of binary mixtures of glucocerebroside and palmitoylcholine phosphatidylethanolamine at physiological temperatures demonstrating that biological membranes have the potential to form quasicrystals, particularly in membrane rafts.¹⁰ Liquid crystals can take a quasiperiodic ordering state.¹¹ Liquid crystals are common in plants with both cellulose and microtubules forming liquid crystalline phases under certain circumstances¹² and DNA itself can generate liquid crystal phases.¹³ These, then, provide other possible avenues for the formation of quasicrystals in plants.

We now return to looking at possible subcellular structures in the development of Fibonacci- and golden ratio-based phyllotaxy. I suggest that quasicrystals may be one way in which subcellular golden ratio-based structures can arise in biological systems since the packing density of pentilings (pentagonal tiling arrays, closely related to aperiodic quasicrystals) approaches $\pi/2$ as pentile number, and similarity to an infinite aperiodic array, increases.¹⁴ As yet it is not clear how this might then translate into phyllotaxy but does demonstrate, for the first time, a mechanism by which the golden ratio might be introduced into biological systems. There is certainly crosstalk between two potential sources of quasicrystals, namely membrane raft¹⁵ and liquid crystals,¹⁶ with mechanical signaling which appears likely to play a major role in plant phyllotaxy.⁶ Transcription factors, which interact with potential quasicrystals in the form of liquid crystal DNA, are also important in phyllotaxy.¹⁷

Other Interesting Properties of Quasicrystals

Recent advances in nanotechnology cast light on the properties of similar materials in biological systems. This includes quasicrystals. Quasicrystals may be useful in

developing powerful computing systems through conventional neurocomputing¹⁸ as well as quantum computing, with the ability of such a system to factorise large integers among other properties, as has been suggested previously.⁹ Quasicrystals also have interesting optical properties. A graded photonic quasicrystal lens has better focusing properties as compared with the graded photonic crystal lens in a frequency range suitable for experimental realization.¹⁹ Thus quasicrystals may play a role in plant structures where optics are important, such as in chloroplasts.

Why are Flowers Beautiful?

In a previous paper I suggested that if plants are conscious this may be in part due to the presence of quasicrystals,⁹ following on from the work of Roger Penrose. Here I suggest that quasicrystals may be common throughout the biological universe and that they may play a role in consciousness in animals, as well as in plants. This means that the golden ratio is necessarily present in the stratum of consciousness wherever it is found. Thus, when we look at a flower or work of art displaying golden ratio-based morphology there may be a resonance with the stratum of our consciousness itself which leads us to regard the flower or work of art as beautiful.

Quasisexual Selection and Aesthetic Selection

Sexual selection is a form of natural selection whereby an organism develops ornamentation that is attractive to the opposite sex in order to maximise reproductive success. It is generally thought that this only occurs among animals. However, there are instances where plants develop flowers which are sexually attractive to animals, for example some species of orchid

and ichneumon wasp. Here the orchid has developed to look like a female wasp, leading to the male wasp to try to mate with it and thus the orchid is pollinated. The orchid is thus not only competing with other orchids for pollination, but also competing with genuine female wasps for the attention of male wasps.

This quasisexual selection may be a subset of a wider form of natural selection used by flowers which I term aesthetic selection. As mentioned above, Fibonacci- and golden ratio-based structures appear beautiful, possibly due to resonance with Fibonacci- and golden ratio-based structures in the nervous system. Plants use flowers constructed using Fibonacci sequences and the golden ratio to attract pollinators through their beauty. Interestingly, recent research on orchids has focused on how selective pressures appear to be relaxed thus enabling the extreme morphologies which appear in flowers.²⁰ Thus aesthetic selection appears to have a similar effect on natural selection in flowers to sexual selection in animals where magnificent structures such as a peacock's tail or a lion's mane develop.

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